Smooth Sailing

How price path and volatility influence momentum's success

A wide body of research, including our own (see **Momentum Works Everywhere**), has established that momentum is a persistent and reliable source of excess return across global equity markets. But could the manner in which momentum manifests – through smooth vs. discrete price movements, or varying levels of volatility – impact its effectiveness? This paper examines how the nature of momentum's path influences its payoff by drawing on academic literature, assessing empirical evidence on how return patterns influence performance, and outlining how we can incorporate this understanding into our investment process.

Key Findings

- **The journey matters.** Stocks with smoother return paths and lower realized volatility show more persistent momentum and stronger risk-adjusted returns across both US and non-US equity universes.
- **Smart stacking of signals.** Composite metrics combining measures for return path smoothness and volatility capture both upside participation and downside protection better than using either signal in isolation, offering a more robust momentum profile.
- **Portfolio construction implications.** Incorporating path characteristics as weighting inputs, rather than exclusionary filters, preserves diversification, delivers similar risk-adjusted performance, and proves more adaptable to real-world portfolios.

Primer

Before diving into the literature, it's helpful to clarify what is meant by the "path" of returns—a concept that underpins the broader question this paper seeks to explore. Two key features characterize this path: its shape and its volatility. The shape refers to how returns accumulate: a continuous profile builds gradually through small, frequent gains or losses (e.g., a stock inching up each day), while a discrete profile reflects infrequent, attention-grabbing price jumps (e.g., a stock spiking 10% on a single earnings day). The volatility, on the other hand, captures how stable or erratic these returns are—whether they fluctuate wildly or progress smoothly, regardless of their directional consistency. While both a smooth, continuous path and a volatile, choppy one might lead to the same cumulative return, the manner in which that return is earned can influence investor reaction, portfolio risk, and ultimately, the strength and persistence of the momentum effect.

Literature Review

One of the most influential studies examining not just whether momentum works but how it manifests, is Da, Gurun, and Warachka's (2014) "*Frog in the Pan[1]: Continuous Information and Momentum*." The authors propose that the form of return accumulation—whether it arrives gradually in small, frequent pieces (continuous information) or in large, infrequent chunks (discrete information)—can impact how investors respond and how momentum behaves. They formalize this idea through the Frog-in-the-Pan (FIP) hypothesis, which posits that investors underreact to a sequence of small signals because such signals fall below their "attention threshold," leading to more persistent and stronger momentum. To test this hypothesis, they construct a metric called Information Discreteness (ID), defined using the signs of daily returns during the momentum formation window. When a stock's past return (PRET) is driven by many small daily moves in the same direction, it receives a low ID score, indicating a continuous information path. Conversely, high ID values reflect past returns being driven by a few large moves, indicating discrete information. They found a momentum premium of 2.40% (post-formation return period of 6 months) by averaging over the winners minus losers, spread across the five ID quintiles. Another interesting finding of Da, Gurun, and Warachka (2014) is that the risk-adjusted momentum profits dissipate much faster for discrete momentum (on average eroded by month 3) compared to that of continuous momentum (on average eroded by month 9). The authors also introduce modified variants of this measure: IDMAG, which weights smaller daily returns more heavily, and IDZ, which adjusts for zero-return days as a proxy for illiquidity. Empirically, they find that stocks with more continuous information (low ID) exhibit significantly stronger and more persistent momentum than those with discrete information (high ID), even when controlling for past returns and standard risk factors. The paper concludes that while continuous information may be less attention-grabbing, investors' persistent underreaction offers a more durable source of excess returns.

Fan, Kearney, Li, and Liu (2022) explore a related but distinct dimension: the volatility of return paths and its impact on momentum effectiveness. Their study, "*Momentum and the Cross-section of Stock Volatility*," identifies realized volatility during the formation period as a critical and often overlooked driver of momentum strategy risk and performance. They observe that standard momentum portfolios (XSMOM), which rank stocks based on cumulative past returns, tend to disproportionately select high-volatility stocks. This concentration contributes to elevated momentum-specific risk and explains the strategy's vulnerability to drawdowns and "momentum crashes" (see **Momentum Crashes: The Long & the Short of It**).

[1] The frog-in-the-pan hypothesis is related to a Greek paradox dating back to the 4th century BCE (the sorites paradox, or paradox of the heap). The premise is that when a frog is placed into a pot of boiling water, the frog will jump out since the dramatic temperature change induces an immediate reaction. However, if the frog is placed in the pot and the water is slowly raised to a boil, the frog will underact and perish. In the context of investing, the hypothesis predicts that investors underreact to small amounts of information that arrives continuously.

To address this, Fan et al. propose a generalized framework for risk-adjusted momentum ranking, termed GRJMOM. Rather than relying on raw past returns, GRJMOM ranks stocks by dividing returns by realized volatility raised to a tunable power, N, allowing flexible control over how aggressively volatility is penalized. When N=0, the strategy collapses to traditional momentum; N=1 approximates a Sharpe ratio sort; N=2 represents a sort over return-to-variance ratios; and higher values of N increasingly tilt the portfolio toward lower-volatility stocks. Across multiple asset classes, they find that momentum profits are strongest in low- and medium-volatility stocks, while high-volatility stocks often exhibit return reversal or insignificant momentum effects. Ultimately, their results highlight that the volatility profile of price paths plays a crucial role in momentum's success and that tailoring momentum signals through a volatility-aware lens can enhance both risk-adjusted returns and downside resilience. This complements the insights from Da et al. (2014) by suggesting that not only the shape of the price path profile (continuous vs. discrete) but also its variability meaningfully affect momentum effectiveness.

Our Own Findings

To evaluate whether the academic insights on return path shape and volatility hold in practice, we sought to replicate key elements of the Da et al. (2014) and Fan et al. (2022) studies across the universes most relevant to our investment strategies. Specifically, we tested whether the Frog-in-the-Pan (FIP) score and risk-adjusted momentum ranking (GRJMOM) could help identify more persistent and resilient momentum across both US and non-US equity universes.

On the US side, we created proxy universes designed to approximate the Russell 3000, 1000, and 2000 indices, with monthly data starting in July 1986 and continuing through June 2025. These universes include publicly traded US companies with price greater than $2, excluding ADRs and secondary listings. The Russell 3000 proxy (all cap) includes the top 3000 companies by market capitalization, the Russell 1000 proxy (large cap) represents the top 1000 companies by market cap, and the Russell 2000 proxy (small cap) is defined as companies ranked 1001-3000 by market cap. For non-US equities, we used the MSCI ACWI ex USA and MSCI ACWI ex USA Small Cap indices as our investable universes, with data beginning in July 2005 and ending June 2025. These universes provided a broad and diversified international counterpart to our US analysis.

To stay consistent with both academic methodology and our internal investment framework, we constructed long-only portfolios using a sequential double-sort process. First, we ranked stocks by their past 12-month return, excluding the most recent month (12-1m formation period). Within the top momentum quintile, we then applied a secondary sort using either the FIP score or the volatility-adjusted momentum signal. Universes were reconstituted annually, while the signal portfolios were rebalanced quarterly and constructed using both equal-weighted and market-cap-weighted approaches.

We evaluated a number of variations on the metrics introduced by Da et al. and Fan et al. but ultimately focused on what we determined to be the most robust signal from each paper:

> **smooth_score:** Based on the original FIP score from Da et al. (2014), this metric captures the shape of return accumulation over the 12-1m formation period. It is calculated as sign(PRET) × (%neg - %pos), where PRET is the cumulative return over the formation window and %pos and %neg represent the proportion of days with positive and negative returns, respectively. Lower scores indicate smoother, more continuous information paths. The range of this measure is [-1,1].
>
> **vol_penalty:** This is our implementation of Fan et al.'s GRJMOM metric, with the volatility penalty parameter tuned to N=2. The signal ranks stocks based on their return-to-variance ratio (i.e. the formation-period return divided by the square of their realized volatility). This penalizes stocks with high return volatility more heavily, allowing us to isolate momentum that is driven by smoother price paths. The range of this measure is all real numbers.

Across all five universes examined, applying an overlay of smooth_score and vol_penalty consistently identified stocks with more durable momentum characteristics. Portfolios constructed using either signal generally delivered stronger risk-adjusted performance than the equal-weighted standard top-quintile 12-1m momentum portfolio (12-1m_momentum_ew) as well as the market-cap weighted variant (12-1m_momentum_mkt). Interestingly, while both signals improved risk-adjusted performance, as reflected in higher Sharpe and Information Ratios, their trade-offs differed. smooth_score delivered stronger upside participation, while vol_penalty exhibited greater downside protection. This complementary behavior suggested potential in combining the two signals[2]. To explore this, we constructed three composite signals using different weightings of smooth_score and vol_penalty:

composite_50_50: an even blend, calculated as (0.5× smooth_score)+(0.5× vol_penalty)

composite_smooth_75: a smooth_score -tilted composite, (0.75× smooth_score)+(0.25× vol_penalty)

composite_vol_75: a vol_penalty -tilted composite, (0.25× smooth_score)+(0.75× vol_penalty)

These blended strategies generally achieved a more balanced profile, capturing a higher share of up-market returns than vol_penalty alone, while preserving much of its downside resilience. In the following table, we use the EXCL_ prefix to denote portfolios constructed using this exclusionary approach: After identifying the top quintile of momentum stocks, we further narrow the selection by purchasing only the top quintile of names based on the respective signal[3] (e.g., smooth_score, vol_penalty, or composite).

[2] By combining signals, we are able to overcome the absence of momentum returns in DOWN markets as found in Galavani (2024).

[3] This quintile-with-in-the-quintile approach selects the top 4% of the universe.

Performance & Risk Statistics – Exclusionary Approach

Russell 1000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	12.96%	2.00%	12.58%	20.70%	0.48	0.16	1.12	1.03
EXCL_vol_penalty_ew	11.87%	0.90%	11.43%	17.72%	0.50	0.08	0.97	0.87
EXCL_composite_50_50_ew	12.63%	1.66%	12.60%	19.36%	0.50	0.13	1.04	0.93
EXCL_composite_smooth_75_ew	12.35%	1.38%	12.76%	20.32%	0.46	0.11	1.08	1.00
EXCL_composite_vol_75_ew	12.27%	1.30%	12.28%	18.44%	0.50	0.11	0.99	0.87
12-1m_momentum_ew	11.94%	0.98%	9.85%	19.63%	0.45	0.10	1.12	1.11
12-1m_momentum_mkt	11.04%	0.08%	9.07%	18.76%	0.43	0.01	1.08	1.10

Russell 3000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	13.75%	2.82%	11.29%	19.90%	0.54	0.25	1.14	1.02
EXCL_vol_penalty_ew	13.32%	2.39%	10.64%	17.72%	0.58	0.22	1.02	0.87
EXCL_composite_50_50_ew	13.93%	3.00%	11.12%	18.71%	0.58	0.27	1.07	0.90
EXCL_composite_smooth_75_ew	13.95%	3.02%	11.11%	19.37%	0.56	0.27	1.11	0.97
EXCL_composite_vol_75_ew	13.70%	2.77%	10.89%	18.14%	0.59	0.25	1.04	0.88
12-1m_momentum_ew	12.24%	1.31%	10.85%	20.69%	0.44	0.12	1.17	1.16
12-1m_momentum_mkt	11.34%	0.41%	9.12%	18.88%	0.44	0.04	1.10	1.11

Russell 2000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	14.58%	4.02%	8.94%	20.94%	0.55	0.45	1.08	0.93
EXCL_vol_penalty_ew	13.65%	3.09%	10.22%	19.06%	0.56	0.30	0.94	0.77
EXCL_composite_50_50_ew	14.66%	4.10%	9.62%	19.62%	0.59	0.43	0.99	0.80
EXCL_composite_smooth_75_ew	14.41%	3.85%	9.22%	20.30%	0.56	0.42	1.04	0.87
EXCL_composite_vol_75_ew	13.84%	3.28%	9.99%	19.12%	0.56	0.33	0.95	0.77
12-1m_momentum_ew	12.19%	1.63%	7.20%	21.69%	0.42	0.23	1.11	1.08
12-1m_momentum_mkt	11.36%	0.80%	8.12%	22.21%	0.37	0.10	1.11	1.11

MSCI ACWI ex-USA Index: July 2005 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	8.47%	5.35%	9.28%	19.35%	0.36	0.58	1.10	0.84
EXCL_vol_penalty_ew	9.28%	6.16%	9.50%	17.33%	0.44	0.65	0.99	0.69
EXCL_composite_50_50_ew	9.16%	6.03%	9.29%	18.15%	0.42	0.65	1.04	0.75
EXCL_composite_smooth_75_ew	8.85%	5.72%	9.04%	18.87%	0.38	0.63	1.09	0.82
EXCL_composite_vol_75_ew	9.05%	5.93%	9.35%	17.59%	0.42	0.63	1.01	0.72
12-1m_momentum_ew	7.57%	4.45%	7.37%	18.89%	0.32	0.60	1.11	0.90
12-1m_momentum_mkt	6.48%	3.36%	8.01%	18.87%	0.26	0.42	1.09	0.94

MSCI ACWI ex-USA Small Cap Index: July 2005 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	12.01%	7.34%	8.02%	19.97%	0.52	0.92	1.16	0.86
EXCL_vol_penalty_ew	11.91%	7.24%	8.80%	18.25%	0.57	0.82	1.06	0.75
EXCL_composite_50_50_ew	11.69%	7.02%	8.48%	18.93%	0.53	0.83	1.10	0.81
EXCL_composite_smooth_75_ew	11.56%	6.89%	8.18%	19.40%	0.51	0.84	1.13	0.84
EXCL_composite_vol_75_ew	11.99%	7.32%	8.83%	18.59%	0.56	0.83	1.08	0.77
12-1m_momentum_ew	9.30%	4.63%	6.96%	19.77%	0.39	0.67	1.13	0.94
12-1m_momentum_mkt	10.07%	5.41%	7.04%	19.58%	0.43	0.77	1.13	0.91

By applying the FIP and GRJMOM methodologies to our investable universes and aligning with our long-only implementation style, we confirmed the central findings of both studies: the path by which momentum is earned—its shape and volatility—has material impact on the payoff profile.

Real-World Implementation

While the exclusionary approach of selecting stocks in the top quintile of momentum and further filtering by return path characteristics proved effective, it came with potential trade-offs. For one, by narrowing the portfolio to only the top quintile of smooth_score or vol_penalty within the top momentum quintile (top ranked 4%), we sacrificed some upside participation. This led us to consider whether a more nuanced application could retain the benefits of path-aware stock selection while preserving broader exposure. Rather than filtering out high-volatility or less continuous names entirely, what if we simply down-weighted them? Drawing from lessons in the academic literature and our own empirical validation, we explored whether applying return path characteristics as weighting inputs rather than binary inclusion/exclusion filters could improve portfolio construction. This approach has the potential to maintain diversification, avoid over-concentration, and capture dynamic momentum profiles (e.g., in sectors like biotech, where returns may be lumpy but information-rich).

In our Russell 1000 proxy, for instance, the exclusionary double-sort approach typically yielded a portfolio of only ~40 names. By applying path-based weights within the full top quintile of momentum (i.e., ~200 stocks), we retain broader exposure. We also avoid potentially excluding stocks that may still contribute positively to performance, even if they exhibit more discrete or volatile return paths.

Applying the same framework as before, we began with the top quintile of momentum but then introduced tiered weighting schemes based on return path scores rather than double-sorting. Within the top momentum quintile, we divided stocks into quintiles based on their smooth_score or vol_penalty values and assigned weights proportional to the square of their tier rank[4]. More precisely, weights were distributed using a quadratic 5^2-to-1^2 scale:

- Quintile 1 (strongest path signal): $5^2 = 25$ weight units
- Quintile 2: $4^2 = 16$ units
- Quintile 3: $3^2 = 9$ units
- Quintile 4: $2^2 = 4$ units
- Quintile 5 (weakest signal): $1^2 = 1$ unit

In the case of the Russell 1000 proxy with 200 names in the top quintile trailing 12-1m, this yielded portfolio weights such as:

- Quintile 1: (45% weight,114 bps per name)
- Quintile 2: (29% weight, 73 bps per name)
- Quintile 3: (16% weight, 41 bps per name)
- Quintile 4: (7% weight, 18 bps per name)
- Quintile 5: (2% weight, 5 bps per name)

[4] In this section we assign equal-weight to all stocks within a given quintile. The quintile breakpoints were computed using Python's pandas.qcut function.

We implemented this weighting approach across the same five universes used in the exclusionary tests, and similar to the section prior, we also created three composite weighting signals from smooth_score and vol_penalty. The weighting approach delivered slightly lower absolute returns than the exclusionary method but achieved comparable Information Ratios, suggesting similar risk-adjusted performance. It maintained much of the upside capture observed in the exclusionary portfolios while still demonstrating meaningful downside resilience—outperforming the standard top quintile 12-1m momentum strategy in most environments, albeit with a bit more give-back during downturns. In the table below, we use the WGT_ prefix to indicate that the signals are applied as weighting schemes, in contrast to their exclusionary (EXCL_) counterparts introduced earlier.

Performance & Risk Statistics – Weighting Schemes

Russell 1000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
WGT_smooth_score	12.15%	1.18%	10.75%	19.82%	0.46	0.11	1.11	1.08
WGT_vol_penalty	12.15%	1.18%	9.85%	18.17%	0.50	0.12	1.04	0.97
WGT_composite_50_50	12.11%	1.14%	10.40%	18.94%	0.48	0.11	1.07	1.02
WGT_composite_smooth_75	12.03%	1.06%	10.63%	19.45%	0.46	0.10	1.09	1.05
WGT_composite_vol_75	12.21%	1.24%	10.11%	18.47%	0.50	0.12	1.05	0.99
12-1m_momentum_ew	11.94%	0.98%	9.85%	19.63%	0.45	0.10	1.12	1.11
12-1m_momentum_mkt	11.04%	0.08%	9.07%	18.76%	0.43	0.01	1.08	1.10

Russell 3000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
WGT_smooth_score	13.29%	2.36%	10.55%	19.90%	0.52	0.22	1.15	1.07
WGT_vol_penalty	13.36%	2.43%	10.09%	18.91%	0.55	0.24	1.10	1.00
WGT_composite_50_50	13.42%	2.49%	10.31%	19.34%	0.54	0.24	1.12	1.03
WGT_composite_smooth_75	13.43%	2.50%	10.42%	19.64%	0.53	0.24	1.14	1.05
WGT_composite_vol_75	13.45%	2.52%	10.21%	19.09%	0.55	0.25	1.11	1.01
12-1m_momentum_ew	12.24%	1.31%	10.85%	20.69%	0.44	0.12	1.17	1.16
12-1m_momentum_mkt	11.34%	0.41%	9.12%	18.88%	0.44	0.04	1.10	1.11

Russell 2000 proxy: July 1986 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
WGT_smooth_score	13.78%	3.22%	7.37%	20.78%	0.52	0.44	1.08	0.97
WGT_vol_penalty	13.90%	3.34%	7.74%	20.02%	0.54	0.43	1.04	0.91
WGT_composite_50_50	13.96%	3.40%	7.58%	20.33%	0.54	0.45	1.06	0.93
WGT_composite_smooth_75	13.80%	3.24%	7.45%	20.58%	0.52	0.43	1.07	0.96
WGT_composite_vol_75	13.84%	3.28%	7.69%	20.13%	0.54	0.43	1.05	0.92
12-1m_momentum_ew	12.19%	1.63%	7.20%	21.69%	0.42	0.23	1.11	1.08
12-1m_momentum_mkt	11.36%	0.80%	8.12%	22.21%	0.37	0.10	1.11	1.11

MSCI ACWI ex-USA Index: July 2005 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
WGT_smooth_score	8.04%	4.91%	7.98%	18.95%	0.34	0.62	1.11	0.88
WGT_vol_penalty	8.35%	5.22%	7.81%	17.88%	0.38	0.67	1.05	0.80
WGT_composite_50_50	8.21%	5.08%	7.92%	18.41%	0.36	0.64	1.08	0.84
WGT_composite_smooth_75	8.26%	5.13%	7.89%	18.74%	0.36	0.65	1.10	0.86
WGT_composite_vol_75	8.16%	5.03%	7.89%	18.10%	0.36	0.64	1.06	0.82
12-1m_momentum_ew	7.57%	4.45%	7.37%	18.89%	0.32	0.60	1.11	0.90
12-1m_momentum_mkt	6.48%	3.36%	8.01%	18.87%	0.26	0.42	1.09	0.94

MSCI ACWI ex-USA Small Cap Index: July 2005 - June 2025

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
WGT_smooth_score	11.11%	6.44%	7.10%	19.70%	0.48	0.91	1.15	0.89
WGT_vol_penalty	11.35%	6.68%	7.44%	18.76%	0.52	0.90	1.10	0.82
WGT_composite_50_50	11.22%	6.55%	7.28%	19.19%	0.50	0.90	1.13	0.86
WGT_composite_smooth_75	11.16%	6.49%	7.15%	19.43%	0.49	0.91	1.14	0.88
WGT_composite_vol_75	11.42%	6.75%	7.42%	18.95%	0.52	0.91	1.11	0.83
12-1m_momentum_ew	9.30%	4.63%	6.96%	19.77%	0.39	0.67	1.13	0.94
12-1m_momentum_mkt	10.07%	5.41%	7.04%	19.58%	0.43	0.77	1.13	0.91

These results demonstrate that weighting path-aware momentum signals offers a compelling middle ground, preserving key performance benefits while addressing real-world portfolio construction challenges. In doing so, this approach brings us closer to an implementation that is not only effective but also scalable and robust.

Conclusion

Momentum is a well-established and enduring source of excess return, but this analysis suggests that how momentum is delivered can meaningfully affect both performance and risk. By examining these return path characteristics through the lens of academic research and applying them within real-world investable universes, we find that there are practical ways to enhance momentum strategies without overhauling them. Whether through thoughtful filtering or dynamic weighting, incorporating return path considerations offers a valuable layer of nuance. These findings open the door to a more refined approach to momentum investing, suggesting meaningful gains may lie in continued research and development. Turns out, in momentum investing, the path taken is just as important as the distance traveled.

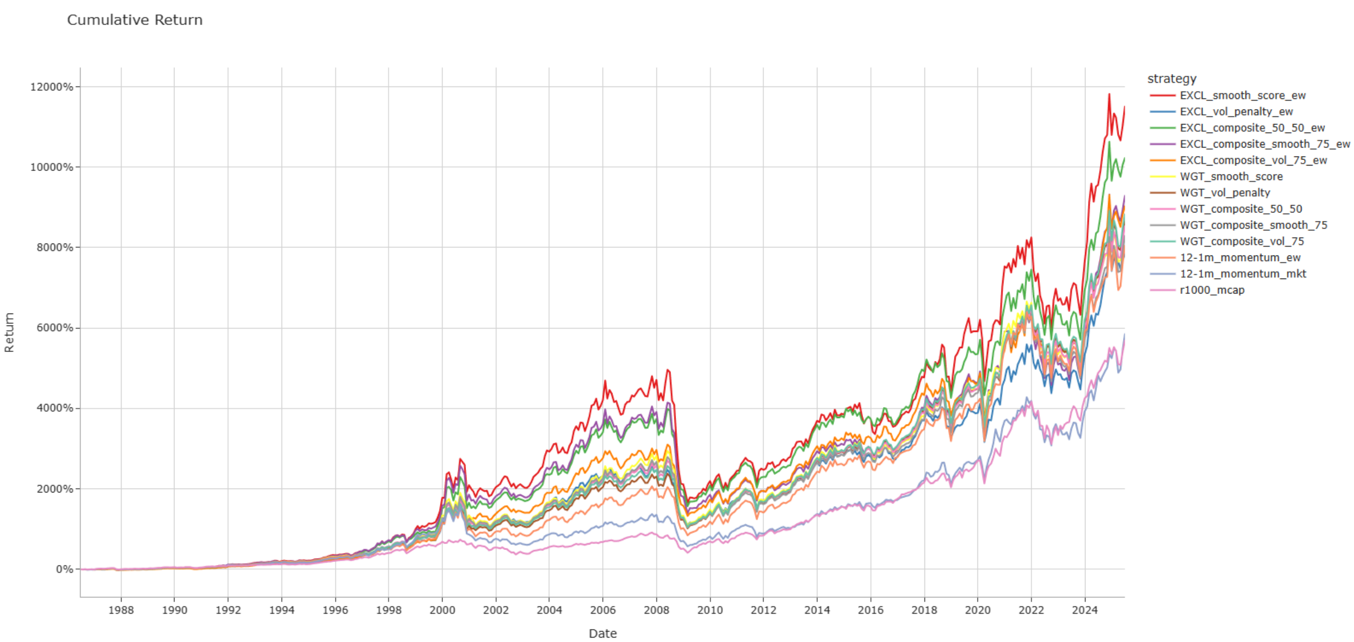
Appendix A

Russell 1000 Index Proxy

A.1 Performance & Risk Statistics

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	12.96%	2.00%	12.58%	20.70%	0.48	0.16	1.12	1.03
EXCL_vol_penalty_ew	11.87%	0.90%	11.43%	17.72%	0.50	0.08	0.97	0.87
EXCL_composite_50_50_ew	12.63%	1.66%	12.60%	19.36%	0.50	0.13	1.04	0.93
EXCL_composite_smooth_75_ew	12.35%	1.38%	12.76%	20.32%	0.46	0.11	1.08	1.00
EXCL_composite_vol_75_ew	12.27%	1.30%	12.28%	18.44%	0.50	0.11	0.99	0.87
WGT_smooth_score	12.15%	1.18%	10.75%	19.82%	0.46	0.11	1.11	1.08
WGT_vol_penalty	12.15%	1.18%	9.85%	18.17%	0.50	0.12	1.04	0.97
WGT_composite_50_50	12.11%	1.14%	10.40%	18.94%	0.48	0.11	1.07	1.02
WGT_composite_smooth_75	12.03%	1.06%	10.63%	19.45%	0.46	0.10	1.09	1.05
WGT_composite_vol_75	12.21%	1.24%	10.11%	18.47%	0.50	0.12	1.05	0.99
12-1m_momentum_ew	11.94%	0.98%	9.85%	19.63%	0.45	0.10	1.12	1.11
12-1m_momentum_mkt	11.04%	0.08%	9.07%	18.76%	0.43	0.01	1.08	1.10

A.2 Cumulative Returns



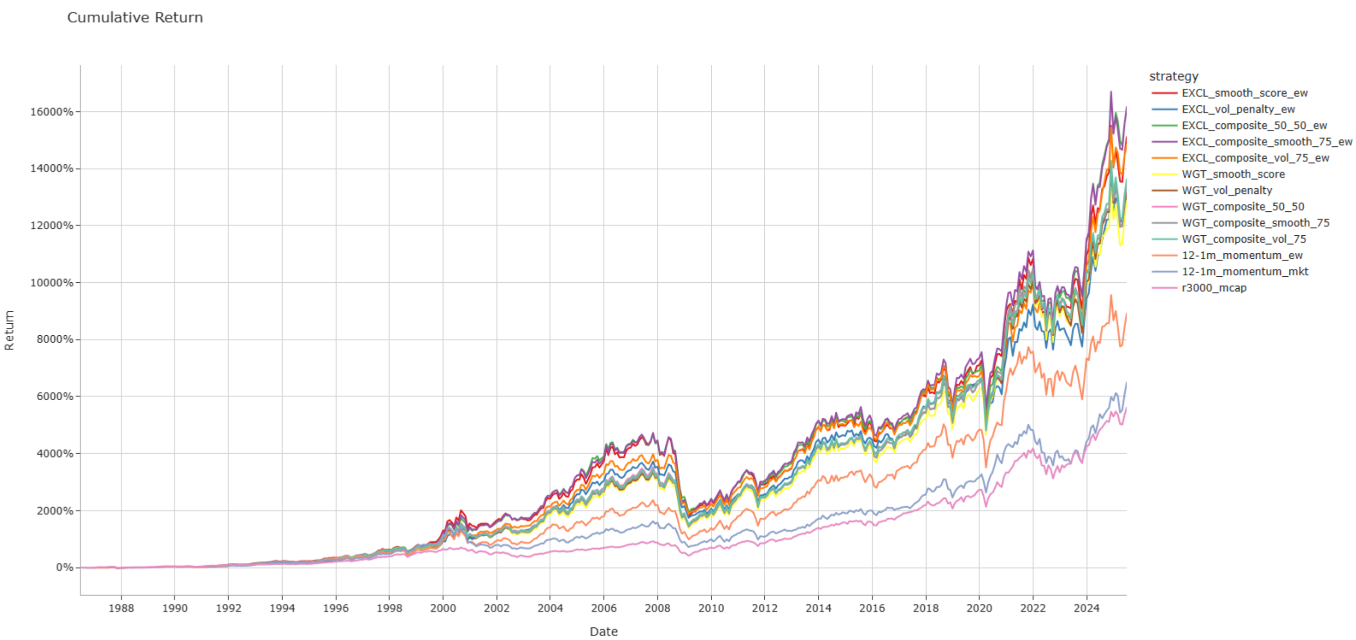
Appendix B

Russell 3000 Index Proxy

B.1 Performance & Risk Statistics

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	13.75%	2.82%	11.29%	19.90%	0.54	0.25	1.14	1.02
EXCL_vol_penalty_ew	13.32%	2.39%	10.64%	17.72%	0.58	0.22	1.02	0.87
EXCL_composite_50_50_ew	13.93%	3.00%	11.12%	18.71%	0.58	0.27	1.07	0.90
EXCL_composite_smooth_75_ew	13.95%	3.02%	11.11%	19.37%	0.56	0.27	1.11	0.97
EXCL_composite_vol_75_ew	13.70%	2.77%	10.89%	18.14%	0.59	0.25	1.04	0.88
WGT_smooth_score	13.29%	2.36%	10.55%	19.90%	0.52	0.22	1.15	1.07
WGT_vol_penalty	13.36%	2.43%	10.09%	18.91%	0.55	0.24	1.10	1.00
WGT_composite_50_50	13.42%	2.49%	10.31%	19.34%	0.54	0.24	1.12	1.03
WGT_composite_smooth_75	13.43%	2.50%	10.42%	19.64%	0.53	0.24	1.14	1.05
WGT_composite_vol_75	13.45%	2.52%	10.21%	19.09%	0.55	0.25	1.11	1.01
12-1m_momentum_ew	12.24%	1.31%	10.85%	20.69%	0.44	0.12	1.17	1.16
12-1m_momentum_mkt	11.34%	0.41%	9.12%	18.88%	0.44	0.04	1.10	1.11

B.2 Cumulative Returns



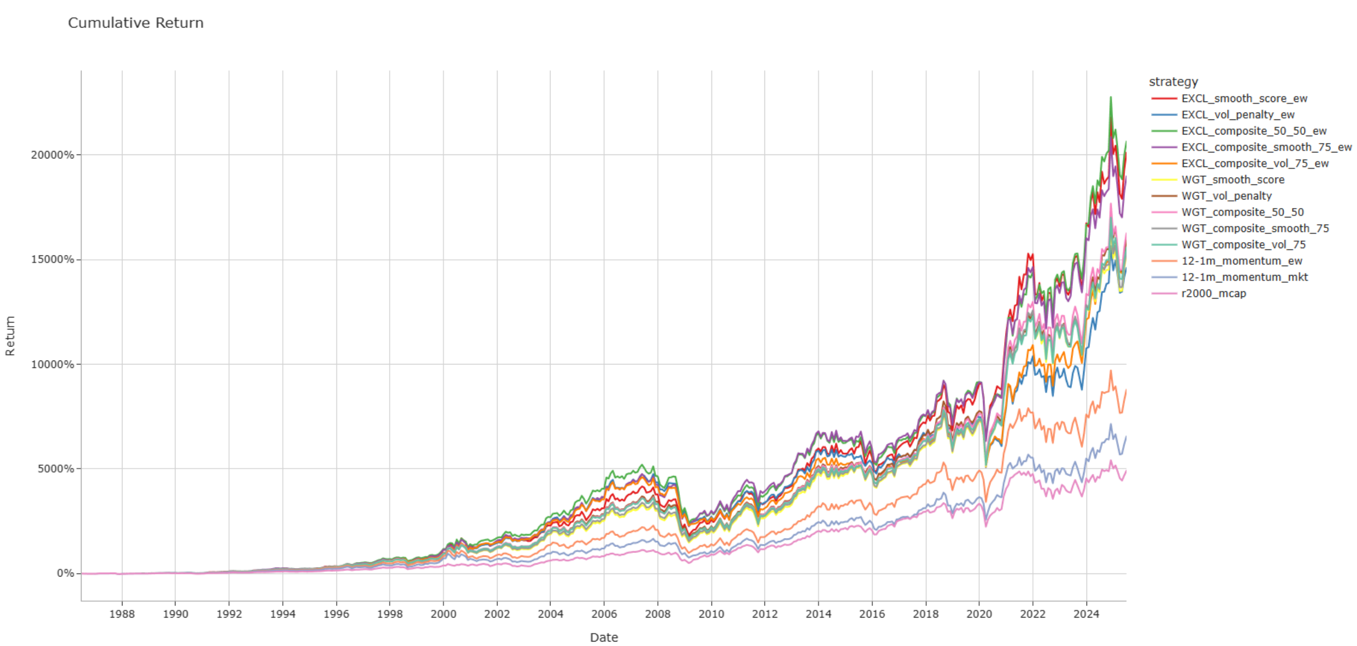
Appendix C

Russell 2000 Index Proxy

C.1 Performance & Risk Statistics

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	14.58%	4.02%	8.94%	20.94%	0.55	0.45	1.08	0.93
EXCL_vol_penalty_ew	13.65%	3.09%	10.22%	19.06%	0.56	0.30	0.94	0.77
EXCL_composite_50_50_ew	14.66%	4.10%	9.62%	19.62%	0.59	0.43	0.99	0.80
EXCL_composite_smooth_75_ew	14.41%	3.85%	9.22%	20.30%	0.56	0.42	1.04	0.87
EXCL_composite_vol_75_ew	13.84%	3.28%	9.99%	19.12%	0.56	0.33	0.95	0.77
WGT_smooth_score	13.78%	3.22%	7.37%	20.78%	0.52	0.44	1.08	0.97
WGT_vol_penalty	13.90%	3.34%	7.74%	20.02%	0.54	0.43	1.04	0.91
WGT_composite_50_50	13.96%	3.40%	7.58%	20.33%	0.54	0.45	1.06	0.93
WGT_composite_smooth_75	13.80%	3.24%	7.45%	20.58%	0.52	0.43	1.07	0.96
WGT_composite_vol_75	13.84%	3.28%	7.69%	20.13%	0.54	0.43	1.05	0.92
12-1m_momentum_ew	12.19%	1.63%	7.20%	21.69%	0.42	0.23	1.11	1.08
12-1m_momentum_mkt	11.36%	0.80%	8.12%	22.21%	0.37	0.10	1.11	1.11

C.2 Cumulative Returns



Appendix D

MSCI ACWI ex-USA Index

D.1 Performance & Risk Statistics

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	8.47%	5.35%	9.28%	19.35%	0.36	0.58	1.10	0.84
EXCL_vol_penalty_ew	9.28%	6.16%	9.50%	17.33%	0.44	0.65	0.99	0.69
EXCL_composite_50_50_ew	9.16%	6.03%	9.29%	18.15%	0.42	0.65	1.04	0.75
EXCL_composite_smooth_75_ew	8.85%	5.72%	9.04%	18.87%	0.38	0.63	1.09	0.82
EXCL_composite_vol_75_ew	9.05%	5.93%	9.35%	17.59%	0.42	0.63	1.01	0.72
WGT_smooth_score	8.04%	4.91%	7.98%	18.95%	0.34	0.62	1.11	0.88
WGT_vol_penalty	8.35%	5.22%	7.81%	17.88%	0.38	0.67	1.05	0.80
WGT_composite_50_50	8.21%	5.08%	7.92%	18.41%	0.36	0.64	1.08	0.84
WGT_composite_smooth_75	8.26%	5.13%	7.89%	18.74%	0.36	0.65	1.10	0.86
WGT_composite_vol_75	8.16%	5.03%	7.89%	18.10%	0.36	0.64	1.06	0.82
12-1m_momentum_ew	7.57%	4.45%	7.37%	18.89%	0.32	0.60	1.11	0.90
12-1m_momentum_mkt	6.48%	3.36%	8.01%	18.87%	0.26	0.42	1.09	0.94

D.2 Cumulative Returns

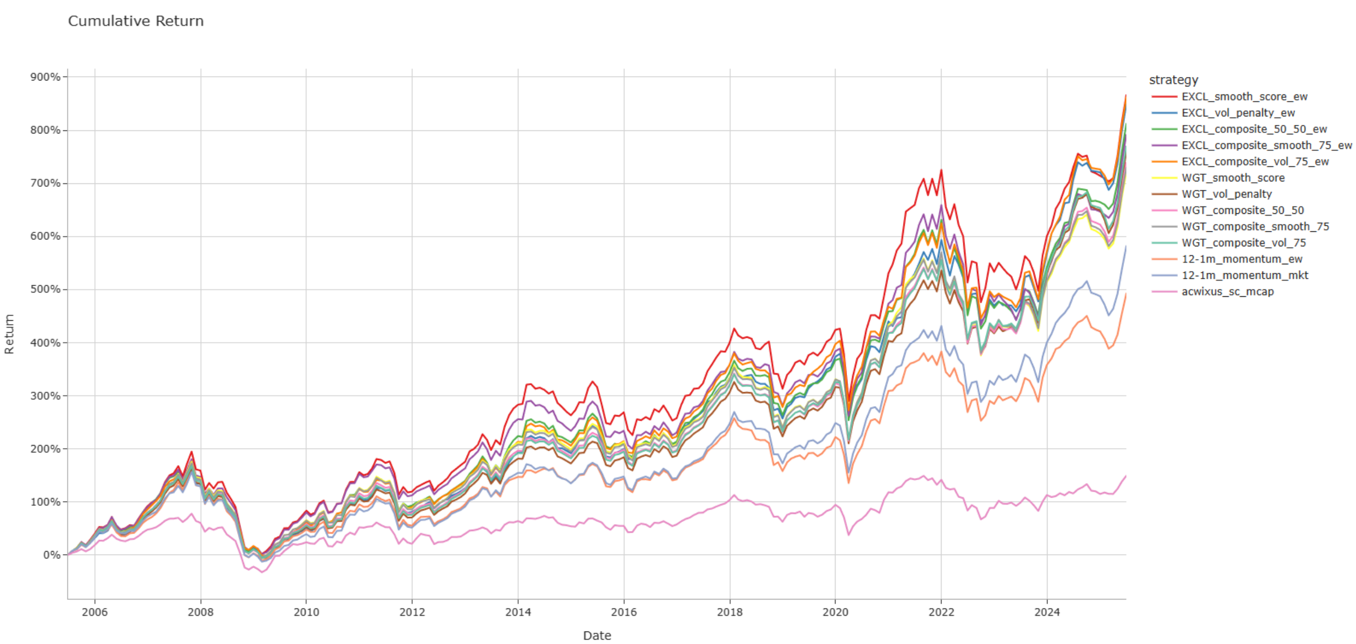
Appendix E

MSCI ACWI ex-USA Small Cap Index

E.1 Performance & Risk Statistics

	Annualized Return	Annualized Excess Return	Tracking Error	Standard Deviation	Sharpe Ratio	Information Ratio	Up Market Capture	Down Market Capture
EXCL_smooth_score_ew	12.01%	7.34%	8.02%	19.97%	0.52	0.92	1.16	0.86
EXCL_vol_penalty_ew	11.91%	7.24%	8.80%	18.25%	0.57	0.82	1.06	0.75
EXCL_composite_50_50_ew	11.69%	7.02%	8.48%	18.93%	0.53	0.83	1.10	0.81
EXCL_composite_smooth_75_ew	11.56%	6.89%	8.18%	19.40%	0.51	0.84	1.13	0.84
EXCL_composite_vol_75_ew	11.99%	7.32%	8.83%	18.59%	0.56	0.83	1.08	0.77
WGT_smooth_score	11.11%	6.44%	7.10%	19.70%	0.48	0.91	1.15	0.89
WGT_vol_penalty	11.35%	6.68%	7.44%	18.76%	0.52	0.90	1.10	0.82
WGT_composite_50_50	11.22%	6.55%	7.28%	19.19%	0.50	0.90	1.13	0.86
WGT_composite_smooth_75	11.16%	6.49%	7.15%	19.43%	0.49	0.91	1.14	0.88
WGT_composite_vol_75	11.42%	6.75%	7.42%	18.95%	0.52	0.91	1.11	0.83
12-1m_momentum_ew	9.30%	4.63%	6.96%	19.77%	0.39	0.67	1.13	0.94
12-1m_momentum_mkt	10.07%	5.41%	7.04%	19.58%	0.43	0.77	1.13	0.91

E.2 Cumulative Returns



Appendix F

Turnover

F.1 Average Annual Turnover

	Russell 1000 Proxy	Russell 3000 Proxy	Russell 2000 Proxy	MSCI ACWI ex-USA	MSCI ACWI ex-USA Small Cap
EXCL_smooth_score_ew	254%	240%	249%	247%	247%
EXCL_vol_penalty_ew	263%	249%	253%	244%	251%
EXCL_composite_50_50_ew	257%	242%	247%	245%	248%
EXCL_composite_smooth_75_ew	253%	239%	247%	244%	247%
EXCL_composite_vol_75_ew	260%	244%	251%	245%	250%
WGT_smooth_score	215%	205%	213%	206%	209%
WGT_vol_penalty	219%	208%	214%	206%	210%
WGT_composite_50_50	215%	204%	212%	203%	207%
WGT_composite_smooth_75	214%	204%	211%	203%	208%
WGT_composite_vol_75	216%	205%	213%	204%	208%
12-1m_momentum_ew	176%	171%	179%	167%	172%
12-1m_momentum_mkt	196%	199%	198%	179%	184%

Appendix G

Signal Selection

G.1 Risk Statistics

To identify the most adequate signal from each paper (Da et al. and Fan et al.), we constructed a proxy investment universe consisting of publicly traded US companies that met the following criteria: positive book value, a share price above $5, and exclusion of ADRs and secondary listings. The analysis was conducted using monthly data beginning in January 2005 and ending April 2025.

In the results below, the baseline_ID signal from Da et al.—which corresponds to our smooth_score—and the ram_2 signal from Fan et al.—aligned with our vol_penalty—emerged as the most suitable strategies. Signal and universe portfolios were rebalanced quarterly, using both equal-weighted and market-cap-weighted methodologies. The exclusionary scheme was applied, whereby the top quintile within the top quintile of the 12–1 month formation period was selected.

	Annualized Return	Standard Deviation	Sharpe Ratio
trailing_12m1_ew	7.70%	19.79%	0.40
baseline_ID_ew	6.62%	19.45%	0.35
magnitude_ID_ew	7.67%	19.37%	0.40
mkt_adj_mag_ID_ew	6.63%	20.19%	0.34
ram_1_ew	7.16%	22.30%	0.35
ram_2_ew	8.54%	17.31%	0.47
ram_3_ew	7.21%	15.72%	0.42
ram_4_ew	7.30%	15.24%	0.44
trailing_12m1_mkt	7.83%	17.43%	0.43
baseline_ID_mkt	8.20%	18.30%	0.44
magnitude_ID_mkt	7.30%	17.67%	0.40
mkt_adj_mag_ID_mkt	9.01%	19.40%	0.46
ram_1_mkt	7.88%	20.35%	0.40
ram_2_mkt	8.21%	16.72%	0.46
ram_3_mkt	8.06%	16.00%	0.47
ram_4_mkt	7.91%	15.53%	0.47

1. trailing_12m1
    a. Top quintile of past 12-month return, excluding the most recent month (12-1m formation period), equal-weighted and market-cap weighted, respectively.
2. baseline_ID
    a. calculated as sign(PRET)x(%neg-%pos]
        i. PRET is defined as a firm's cumulative return over the past twelve months after skipping the most recent month.
        ii. The percentage of days during the formation period with positive and negative returns are denoted %pos and %neg.
3. magnitude_ID
    a. Takes the baseline_ID score and incorporates the magnitude of daily returns by assigning more weight to small daily returns. This follows the assumption that many small daily gains are more representative of a continuous price path.
4. mkt_adj_mag_ID
    a. An iteration of the Magnitude ID score that adjusts the daily returns by subtracting value-weighted market returns.
5. ram_N
    a. ranks stocks by dividing returns by realized volatility raised to a tunable power, N, allowing flexible control over how aggressively volatility is penalized. When N=0, the strategy collapses to traditional momentum; N=1 approximates a Sharpe ratio sort; N=2 represents a sort over return-to-variance ratios; and higher values of N increasingly tilt the portfolio toward lower-volatility stocks.

Note: the _ew suffix denotes an equal-weighted portfolio, while _mkt denotes market-cap weighting.

Appendix H

Risk Statistics Formulas

H1. Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{\text{Annualized Return} - \text{Annualized Risk-Free Rate}}{\text{Annualized Volatility}}$$

H2. Information Ratio

$$\text{Information Ratio} = \frac{\text{Annualized Excess Return}}{\text{Tracking Error}}$$

H.3 Market Capture Ratios

$$\text{Up Capture} = \frac{\text{Average of } r_{\text{strategy}} \text{ when } r_{\text{benchmark}} > 0}{\text{Average of } r_{\text{benchmark}} \text{ when } r_{\text{benchmark}} > 0}$$

$$\text{Down Capture} = \frac{\text{Average of } r_{\text{strategy}} \text{ when } r_{\text{benchmark}} < 0}{\text{Average of } r_{\text{benchmark}} \text{ when } r_{\text{benchmark}} < 0}$$

About IMC

IMC is solely focused on helping clients build better portfolios through our Informed Momentum investment approach. This approach has been applied consistently across all strategies since the inception of the firm in 2007. The daily application of our systematic process is designed to deliver consistent and predictable results. Since our entire company works for a single objective, it only makes sense to align the name of our brand with exactly what we do every day.

We are the **Informed Momentum Company.**

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Important Disclosures: